

Cosmological vector modes and quantum gravity effects

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Abstract

In contrast to scalar and tensor modes, vector modes of linear perturbations around an expanding Friedmann–Robertson–Walker universe decay. This makes them largely irrelevant for late time cosmology, assuming that all modes started out at a similar magnitude at some early stage. By now, however, bouncing models are frequently considered which exhibit a collapsing phase. Before this phase reaches a minimum size and re-expands, vector modes grow. Such modes are thus relevant for the bounce and may even signal the breakdown of perturbation theory if the growth is too strong. Here, a gauge invariant formulation of vector mode perturbations in Hamiltonian cosmology is presented. This lays out a framework for studying possible canonical quantum gravity effects, such as those of loop quantum gravity, at an effective level. As an explicit example, typical quantum corrections, namely those coming from inverse densitized triad components and holonomies, are shown to increase the growth rate of vector perturbations in the contracting phase, but only slightly. Effects at the bounce of the background geometry can, however, be much stronger.

1 Introduction

Linear perturbations on isotropic expanding or contracting geometries can be split in different types according to their transformation properties under spatial rotations: scalar, vector and tensor modes. In general, this presents a convenient decomposition of general perturbations into different classes. But it acquires a more important role for the linearized Einstein's equation where the different modes decouple. Moreover, linearized gauge transformations corresponding to changes of space-time coordinates do not mix the modes which can thus be analyzed separately. In this article we focus on the vector mode and possible quantum effects in its evolution.

Vector mode perturbations in classical cosmology decay in an expanding universe, and dynamically become of less interest as the universe continues its expansion. Accordingly,

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vector modes are often ignored. This feature holds true for expanding cosmologies which start from a big bang or emerge from a quantum state if one assumes all modes to be of comparable initial magnitude. However, this assumption has to be justified. One possibility is to use bouncing models where the pre-history before the big bang is described by a collapsing phase. Then, vector perturbations can lead to significant problems because gauge invariant measures of vector perturbations grow. Their current size relative to that of scalar and tensor modes then depends on where equal sizes or other initial conditions are assumed. Moreover, vector modes are generated by higher order perturbations and subsequently grow in a contracting phase [1]. They can thus not be ignored altogether.

There are several models such as string inspired pre-big bang scenarios [2, 3] or cyclic and ekpyrotic models [4, 5], or some models of loop quantum cosmology [6, 7] which exhibit a bounce. Such scenarios for an avoidance of the big bang singularity are developed mainly based on homogeneous models of cosmology. On the other hand, the growth of vector perturbations in the contracting phase indicates a possible violation of the homogeneity assumption when the bounce is approached as indicated by the breakdown of classical perturbation theory. Thus, the growth of vector perturbations not only raises questions regarding the validity of an homogeneity assumption but may even question the phenomena of the bounce itself.¹

In this paper, we derive the vector mode dynamics in the context of cosmological models based on loop quantum gravity [11, 12, 13] and cosmology [14]. Firstly, we present a systematic derivation of classical vector perturbation equations using a canonical formulation in Ashtekar variables [15, 16]. We compute the gauge transformations property for the vector perturbations and then construct the corresponding gauge invariant variable. All this is done in a purely canonical way to outline the general procedure followed also in the presence of quantum corrections.

In the following section we study possible effects of quantum corrections expected from loop quantum gravity. We compute requirements on quantum correction functions from anomaly cancellation in the quantum corrected constraint algebra. These expressions include inhomogeneities in such a way that all symmetries of a Friedmann–Robertson–Walker background are broken. Thus, we are not dealing with a mini-superspace quantization even though inhomogeneities are restricted to the perturbative vector mode. How individual terms of effective constraints are related to operators in the full theory is described in [17]. We study the effects of quantum correction arising for inverse densitized triads in detail, and perform corresponding calculations for corrections from holonomies in a subsequent section. While current methods are not sufficient to compute full correction functions for all gauges, it turns out that the remaining freedom is constrained by requiring the absence of anomalies as a consistency condition. This provides evidence, at the effective level employed here, that anomaly cancellation will restrict possible quantization choices of the full

¹Based on matching conditions, an evolution of vector modes through a bounce has been studied, e.g., in [8, 9], assuming certain forms of a non-singular bouncing background geometry. This does not address the validity of a perturbativity assumption. Vector modes have occasionally been considered in inflationary scenarios such as in [10]. Since this happens in an expanding phase, vector modes decay and do not challenge the perturbativity assumption either.

theory. (See [18, 19] for analogous statements based on different principles.) Moreover, we demonstrate explicitly that anomaly-free set of constraints, and thus a covariant effective space-time picture, is possible even in the presence of non-trivial quantum corrections. As one application, we show that in perturbative regimes (not close to a bounce) quantum corrections make the growth rate of the vector mode in a contracting universe slightly stronger compared to the classical behavior.

2 Canonical formulation

In this paper, we study the vector mode of linear metric perturbations around spatially flat Friedmann-Robertson-Walker (FRW) spacetimes. The procedure we use is analogous to that for scalar modes [20], although specifics certainly change when vector modes are considered. The general form of the perturbed metric around the FRW background containing only the vector mode is given by

$$g_{00} = -a^2 \quad , \quad g_{0a} = a^2 S_a \quad , \quad g_{ab} = a^2 [\delta_{ab} + F_{a,b} + F_{b,a}] \quad . \quad (1)$$

The perturbation fields F^a and S^a satisfy $F^a{}_{,a} = 0$ and $S^a{}_{,a} = 0$ to separate them from scalar gradient terms. In other words these divergence free fields describe the vorticity of metric perturbations. An index 0 refers to conformal time η , while a, b, \dots refer to co-moving spatial coordinates. In a canonical formulation, the distance element is expressed in terms of the spatial metric q_{ab} , the lapse function N and the shift vector N^a , related to the spacetime metric through

$$g_{00} = -N^2 + q_{ab} N^a N^b \quad , \quad g_{0a} = q_{ab} N^b \quad , \quad g_{ab} = q_{ab} \quad . \quad (2)$$

By comparing expression (1) for a perturbed spacetime metric to the relation (2), one can see that in canonical formulations the vector perturbation is generated through the perturbations of the shift vector N^a and off-diagonal components of the spatial metric q_{ab} . In particular, the lapse function, being scalar, does not contribute to vector mode dynamics.

2.1 Background

In canonical gravity, the spatial metric q_{ab} plays the role of a configuration variable with momenta related to extrinsic curvature

$$K_{ab} = \frac{1}{2N} (\dot{q}_{ab} - 2D_{(a} N_{b)}) \quad (3)$$

in terms of a spatial covariant derivative D_a . The lapse function N and shift vector N^a do not have momenta and are not dynamical since they do not appear as time derivatives in the action. They rather play the role of multipliers to constraints which will be written explicitly below for the vector mode.

However, in view of including quantum gravity effects we do not use metric variables but connection variables which follow after a canonical transformation and provide the basis for a loop quantization of gravity [21]. In these Ashtekar variables [15, 16] one uses a densitized triad E_i^a instead of the spatial metric, defined as

$$E_i^a := |\det(e_a^i)| e_i^a \quad (4)$$

in terms of the co-triad e_a^i and its inverse e_i^a which in turn are defined via the spatial metric as $q_{ab} =: e_a^i e_b^i$. The canonically conjugate variable to the densitized triad is the Ashtekar connection $A_a^i := \Gamma_a^i + \gamma K_a^i$, where K_a^i is extrinsic curvature and $\gamma > 0$ is the so-called Barbero-Immirzi parameter [16, 22]. The spin connection Γ_a^i by definition satisfies $D_a e_i^a = 0$ which can be solved as

$$\Gamma_a^i = -\epsilon^{ijk} e_j^b (\partial_{[a} e_{b]}^k + \frac{1}{2} e_k^c e_a^l \partial_{[c} e_{b]}^l) \quad (5)$$

As mentioned, we perturb basic variables around a spatially-flat FRW background. Further, we denote background variables with a bar as in

$$\bar{E}_i^a = \bar{p} \delta_i^a \quad , \quad \bar{\Gamma}_a^i = 0 \quad , \quad \bar{K}_a^i = \bar{k} \delta_a^i \quad , \quad \bar{N} = \sqrt{\bar{p}} \quad , \quad \bar{N}^a = 0 \quad , \quad (6)$$

where $\bar{p} = a^2$ and the spatial metric is $\bar{q}_{ab} = a^2 \delta_{ab}$. (In general, \bar{p} as a triad rather than metric component could be negative, which however we can safely ignore here.) The choice of $\bar{N} = a$ leads to conformal time. One may emphasize here that for a spatially flat FRW spacetime the background spin connection $\bar{\Gamma}_a^i$ vanishes, thus $\bar{A}_a^i = \gamma \bar{K}_a^i = \gamma \bar{k} \delta_a^i$. Moreover, $\bar{k} = da/d\eta = \dot{a}/a$ in conformal time.

2.2 Perturbed canonical variables

The perturbed densitized triad and Ashtekar connection around a spatially flat FRW background are given by

$$E_i^a = \bar{p} \delta_i^a + \delta E_i^a \quad , \quad A_a^i = \Gamma_a^i + \gamma K_a^i = \gamma \bar{k} \delta_a^i + (\delta \Gamma_a^i + \gamma \delta K_a^i) \quad , \quad (7)$$

where \bar{p} and $\gamma \bar{k}$ are the background densitized triad and Ashtekar connection. At the linear level, the general solution for the co-triad corresponding to $q_{ab} = a^2(\delta_{ab} + F_{a,b} + F_{b,a})$ as in equation (1) is

$$e_a^i = a [\delta_a^i + (c_1 F_{a,}{}^i + c_2 F^i{}_{,a})] \quad , \quad (8)$$

where $c_1 + c_2 = 1$. Specific values of c_1 and c_2 are part of the gauge choice of the triad as a set of three vector fields in arbitrary rotation. Using definition (4) for the densitized triad one can easily compute the expression of the perturbed densitized triad for vector mode

$$\delta E_i^a = -\bar{p} (c_1 F_{i,}{}^a + c_2 F^a{}_{,i}) \quad , \quad (9)$$

where we have used the divergence free property, i.e. $\delta_a^i \delta E_i^a = 0$, for vector mode perturbations; thus no linear term results from the determinant used in defining the densitized

triad. By comparing the perturbed spacetime metric from expression (1) with the relation (2), one can read off the expression for the perturbed lapse function and shift vector

$$\delta N = 0 \quad , \quad \delta N^a = S^a . \quad (10)$$

Using the spin connection (5) and the general expression of the perturbed densitized triad in (7), the linearized spin connection is given by

$$\delta \Gamma_a^i = \frac{1}{\bar{p}} \epsilon^{ije} \delta_{ac} \partial_e \delta E_j^c . \quad (11)$$

2.3 Canonical structure of linearized vector modes

In a canonical formulation, the Einstein-Hilbert action can be written equivalently (up to boundary terms) using the Ashtekar connection and densitized triad as

$$S_{EH} = \int dt \int_{\Sigma} d^3x \left[\frac{1}{8\pi G \gamma} E_i^a \mathcal{L}_t A_a^i - [\Lambda^i G_i + N^a \mathcal{C}_a + N \mathcal{C}] \right] , \quad (12)$$

where Λ^i , N^a and N are Lagrange multipliers of the Gauss, diffeomorphism and Hamiltonian constraints, explicit expressions of which are written below. Before decomposing the symplectic structure according to (7), we introduce a cell to render the homogeneous mode well-defined. Integrating the first term of (12) only over a finite box of coordinate volume V_0 with perturbed basic variables of the form (7), we obtain the symplectic structure given by the Poisson brackets of the background and perturbed variables as

$$\{\bar{k}, \bar{p}\} = \frac{8\pi G}{3V_0} \quad , \quad \{\delta K_a^i(x), \delta E_j^b(y)\} = 8\pi G \delta^3(x, y) \delta_a^b \delta_j^i . \quad (13)$$

In deriving these relation, we have used the properties that for vector perturbations $\delta_a^i \delta E_i^a = \delta_i^a \delta K_a^i = 0$. This provides separate canonical structures for the background and perturbations, but these variables will be coupled dynamically. In particular, the homogeneous background is dynamical and would receive back-reaction effects at higher than linear orders.

2.3.1 Gauss constraint

In triad variables, a Gauss constraint appears which generates internal gauge rotations of phase space functions because triads whose legs are rotated at a fixed point correspond to the same spatial metric. This constraint is given by

$$G(\Lambda) := \int_{\Sigma} d^3x \Lambda^i G_i = \frac{1}{8\pi G \gamma} \int_{\Sigma} d^3x \Lambda^i (\partial_a E_i^a + \epsilon_{ij}{}^k A_a^j E_k^a) . \quad (14)$$

Using the perturbed form of basic variables (7), it can be reduced to

$$G(\Lambda) = \frac{1}{8\pi G} \int_{\Sigma} d^3x \Lambda^i (\epsilon_{ij}{}^a \bar{p} \delta K_a^j + \epsilon_{ia}{}^k \bar{k} \delta E_k^a) . \quad (15)$$

Since we are working with a background $E_i^a = \bar{p}\delta_i^a$ whose gauge freedom is fixed, the multiplier Λ^i is already of first order. To derive the expression (15), we have used the definition of the spin connection Γ_a^i in terms of the densitized triad E_i^a , which reduces to (11) for the linearized equations. Internal gauge rotations of phase space functions are parametrized by the Lagrange multiplier Λ^i through $\delta_\Lambda f = \{f, G[\Lambda]\}$. In particular, the internal gauge rotations of perturbed basic variables are

$$\delta_\Lambda(\delta K_a^i) := \{\delta K_a^i, G(\Lambda)\} = \bar{k}\Lambda^l \epsilon_{la}{}^i \quad , \quad \delta_\Lambda(\delta E_i^a) := \{\delta E_i^a, G(\Lambda)\} = -\bar{p}\Lambda^l \epsilon_{li}{}^a \quad . \quad (16)$$

Clearly, the perturbed variables themselves are not invariant under internal gauge rotations in spite of the fixed background. However, one may notice already that the symmetrized perturbed variables are in fact invariant under internal gauge rotations. Thus, the physical quantities must depend only on the symmetrized form of the perturbed basic variables. Then, the constants c_1 and c_2 in (9) are irrelevant since

$$E_{(i}{}^{a)} = -\frac{1}{2}\bar{p}(F_{i,}{}^a + F^a{}_{,i}) \quad .$$

2.3.2 Diffeomorphism constraint

The diffeomorphism constraint generates gauge transformations corresponding to spatial coordinate transformations of phase space functions. Its general contribution from gravitational variables is given by

$$D_G[N^a] := \int_\Sigma d^3x N^a \mathcal{C}_a = \frac{1}{8\pi G \gamma} \int_\Sigma d^3x N^a [F_{ab}^i E_i^b - A_a^i G_i] \quad (17)$$

(where the subscript “G” stands for “gravity” to separate the term from the matter contribution. In general, a matter field would also contribute a term D_m to the diffeomorphism constraint, which we leave unspecified here and relate later to the stress-energy tensor.) The second term vanishes by virtue of the Gauss constraint, but is necessary to generate diffeomorphisms in the form of Lie derivatives of phase space functions along the shift vector. Using the expression of the perturbed basic variables (7), one can reduce the diffeomorphism constraint to

$$D_G[N^a] = \frac{1}{8\pi G} \int_\Sigma d^3x \delta N^c [-\bar{p}(\partial_k \delta K_c^k) - \bar{k} \delta_c^k (\partial_d \delta E_k^d)] \quad . \quad (18)$$

Here, we have kept up to quadratic terms in the perturbations, noting $\bar{N}^a = 0$.

2.3.3 Hamiltonian constraint

In a canonical formulation, the Hamiltonian constraint generates ‘time evolution’ of the spatial manifold for phase space functions satisfying the equations of motion. Its gravitational contribution in Ashtekar variables is

$$H_G[N] = \frac{1}{16\pi G} \int_\Sigma d^3x N \frac{E_j^c E_k^d}{\sqrt{|\det E|}} \left(\epsilon_i{}^{jk} F_{cd}^i - 2(1 + \gamma^2) K_{[c}^j K_{d]}^k \right) \quad . \quad (19)$$

Using the general perturbed forms of basic variables and the expression of curvature $F_{ab}^i = \partial_a A_b^i - \partial_b A_a^i + \epsilon^i_{jk} A_a^j A_b^k$, one can simplify (19). Up to quadratic terms it is given by

$$H_G[N] = \frac{1}{16\pi G} \int_{\Sigma} d^3x \bar{N} \left[-6\bar{k}^2 \sqrt{\bar{p}} - \frac{\bar{k}^2}{2\bar{p}^{\frac{3}{2}}} (\delta E_j^c \delta E_k^d \delta_c^k \delta_d^j) \right. \\ \left. + \sqrt{\bar{p}} (\delta K_c^j \delta K_d^k \delta_k^c \delta_j^d) - \frac{2\bar{k}}{\sqrt{\bar{p}}} (\delta E_j^c \delta K_c^j) \right] \quad (20)$$

with $\delta N = 0$ for vector modes. We may mention here that γ dependent terms drop out of the Hamiltonian constraint once the spin connection, the generic form (9) of the densitized triad and the extrinsic curvature are used. Again, there is a matter contribution H_m left unspecified here.

2.4 Gauge transformations and gauge invariant variables

General gauge transformations are determined by a choice of N and N^a , which gives rise to all space-time coordinate transformations. For the vector mode, only the choice $N^a = \xi^a$ with $\xi^a_{,a} = 0$ is relevant since the remaining functions N or $N_a = \xi_{,a}$ would affect only the scalar mode. Thus, transformations of interest here are generated only by diffeomorphism constraint in a form parametrized by the shift vector as $\delta_{\xi} f = \{f, D_G[N^a = \xi^a]\}$. We need to consider this only to linear order, using the linearized constraints. The resulting transformation for basic variables will then be used to find gauge invariant combinations. Alternatively, a canonical formulation allows one to compute gauge invariant observables first, and then linearize [23, 24].

Under such a gauge transformation, the perturbed densitized triad and extrinsic curvature transform as

$$\delta_{\xi}(\delta E_i^a) = -\bar{p} \partial_i \xi^a \quad , \quad \delta_{\xi}(\delta K_a^i) = \bar{k} \partial_a \xi^i \quad . \quad (21)$$

Relating δE_i^a to δF^a and δK_a^i to δS^a based on (3) leads to gauge transformations for the vector mode functions F^a and S^a :

$$\delta_{\xi} F^a = \xi^a \quad , \quad \delta_{\xi} S^a = \dot{\xi}^a \quad , \quad \delta_{\xi} \sigma^a = \delta_{\xi} (S^a - \dot{F}^a) = 0 \quad (22)$$

introducing $\sigma^a := S^a - \dot{F}^a$ as a gauge invariant variable for the gravitational vector mode.

2.5 Linearized equations of motion

General equations of motion are written canonically as

$$\dot{f} = \{f, \mathcal{H}\} \quad (23)$$

for any phase space function f using the total Hamiltonian \mathcal{H} . For gravity, $\mathcal{H} = H_G[N] + D_G[N^c] + H_m[N] + D_m[N^c]$ with the gravitational contributions H_G and D_G and matter contributions H_m and D_m to the Hamiltonian and diffeomorphism constraints. Equations

of motion refer to coordinate time, with derivatives being indicated by the dot. The form of the lapse function N specifies which time coordinate is used; here, $\bar{N} = a = \sqrt{\bar{p}}$ implies conformal time η . The general form (23) also applies to equations of motion for momenta of the multipliers, such as $\dot{P}_N = 0 = \{P_N, \mathcal{H}\} = -\delta\mathcal{H}/\delta N$. This must be zero because the momentum P_N is zero for an action (12) not depending on the time derivative of N . In this way, equations of motion for the momenta of lapse and shift give rise to constraints.

Hamilton's equation of motion for the perturbed densitized triad

$$\delta \dot{E}_i^a = \{\delta E_i^a, \mathcal{H}\} \quad (24)$$

leads to the expression (3) of extrinsic curvature, but linearized. We will only need it in symmetrized form which is

$$\delta K_{(a}{}^{i)} = \frac{1}{2} \left[\bar{k}(F_{a,}{}^i + F^i{}_{,a}) + (\dot{F}_{a,}{}^i + \dot{F}^i{}_{,a}) - (S_{a,}{}^i + S^i{}_{,a}) \right] . \quad (25)$$

or

$$\delta K_{(a}{}^{i)} = -\frac{1}{2}(\sigma_{a,}{}^i + \sigma^i{}_{,a}) + \frac{1}{2}\bar{k}(F_{a,}{}^i + F^i{}_{,a}) \quad (26)$$

using the gauge-invariant variable σ^a . Variation with respect to the shift vector δN^a ,

$$\frac{\delta}{\delta(\delta N^a)} \mathcal{H} = 0 , \quad (27)$$

when expressed in terms of symmetrized vector perturbations implies

$$-\frac{\bar{p}}{2} \nabla^2 \sigma_a = 8\pi G \left[\frac{\delta D_m}{\delta(\delta N^a)} \right] . \quad (28)$$

Using the relation between the perturbed stress-energy tensor and a variation of the matter diffeomorphism constraint D_m with respect to the shift vector as derived in [20],

$$-\frac{1}{\bar{N}\bar{p}^{3/2}} \left[\frac{\delta D_m}{\delta(\delta N^a)} \right] = \delta T^{(v)0}{}_a =: -(\rho + P)V_a . \quad (29)$$

one can express equation (28) in standard form [25]

$$-\frac{1}{2a^2} \nabla^2 \sigma_a = 8\pi G(\rho + P)V_a \quad (30)$$

for a vector mode equation. Here ρ and P are energy density and pressure of the background matter field.

The second vector mode equation comes from Hamilton's equation of motion for perturbed extrinsic curvature, which using (26) becomes

$$-\frac{1}{2} \frac{d}{d\eta} (\sigma_{a,}{}^i + \sigma^i{}_{,a}) - \bar{k}(\sigma_{a,}{}^i + \sigma^i{}_{,a}) = 8\pi G \bar{p} \delta T^{(v)i}{}_{(a)} . \quad (31)$$

The perturbed spatial stress tensor for the vector mode in terms of the matter Hamiltonian [20] is

$$\delta T^{(v)(i)}_{a} = \frac{1}{\bar{p}} \left[\frac{1}{3V_0} \frac{\partial H_m}{\partial \bar{p}} \left(\frac{\delta E_j^c \delta_{(a}^j \delta_{c}^{i)}}{\bar{p}} \right) + \frac{\delta H_m}{\delta(\delta E^{(a)}_{i)}} \right]. \quad (32)$$

By using the expression of the perturbed stress-energy tensor in terms of anisotropic stress π_a as $\delta T^{(v)(i)}_{a} =: P(\pi_a,{}^i + \pi^i,{}_a)$ we can express equation (31) in standard form [25]

$$-\frac{1}{2a^4} \frac{d}{d\eta} [a^2(\sigma_a,{}^i + \sigma^i,{}_a)] = 8\pi G P(\pi_a,{}^i + \pi^i,{}_a) \quad (33)$$

for the second vector mode equation.

For vector modes, variation with respect to the lapse function does not give new field equations but would rather contribute back-reaction terms to the background evolution. The two equations (28) and (31) thus provide the complete dynamics.

3 Quantum dynamics: Inverse triad corrections

We have completed the derivation of vector mode equations in Hamiltonian cosmology based on Ashtekar's formulation of general relativity. This naturally agrees with results of [25]. As mentioned, we are interested in applying this formulation to study possible canonical quantum gravity effects. As an explicit example, we now consider quantum corrections coming from terms containing inverse densitized triad components. In Sec. 4 we provide formulas for a second major quantum correction that one expects from the use of holonomies, rather than direct connection components, as basic operators of the quantum theory. This has the effect of adding terms of higher order in extrinsic curvature components, and thus higher powers of the first time derivative of the metric, to the Hamiltonian. In addition to those two corrections, there are higher derivative corrections implied by genuine quantum effects [26, 27]. All this combines to effective constraints or effective equations of motion for the system. As for the diffeomorphism constraint, we assume that it receives no quantum corrections because it is quantized directly through its phase space transformations [28].

3.1 Quantum corrected Hamiltonian constraint

While homogeneous quantum cosmology using loop quantum gravity techniques is rather well understood [14], a systematic derivation of quantum corrections to classical dynamics which includes inhomogeneity is not yet available. But typical effects are known and provide valuable indications for implications of quantum effects. In loop quantum gravity, the appearance of inverse powers of the densitized triad as in $\frac{E_j^c E_k^d}{\sqrt{|\det E|}}$ initially leads to difficulties since flux operators quantizing the densitized triad have discrete spectra containing zero as an eigenvalue [29, 30]. These difficulties can be overcome in a way exploiting

background independence, and giving rise to well-defined operators [31, 32]. However, for small values of the densitized triad, where the classical expression would diverge, there are deviations from the classical behavior which imply quantum corrections. For the homogeneous case, explicit calculations show that the classical term is multiplied by a factor $\bar{\alpha}$ [33] making the whole expression finite. For large fluxes, the leading terms in explicit expressions of $\bar{\alpha}$ are of the form

$$\bar{\alpha}(\bar{p}) = 1 + c \left(\frac{\ell_P^2}{\bar{p}} \right)^n, \quad (34)$$

where n and c are positive definite numbers which correspond to a given inverse triad operator. However, they are not completely fixed since triad operators themselves are subject to quantization ambiguities [34, 35]. One important motivation to study inhomogeneous models is that, compared to homogeneous models, their dynamics gives rise to much tighter consistency conditions which could constrain such parameters. This is indeed borne out, as we will see later. While general derivations of α for inhomogeneous configurations without assumptions on the metric such as symmetries or specific modes is complicated (see e.g. [36]) and does not yet provide many insights, α can be computed in certain perturbative regimes. It has been studied recently for scalar mode perturbations [20], showing a similar behavior as in the isotropic case. Here we consider such quantum corrections for vector mode perturbations, although this is a case where no explicit expression is available yet. As we will see, consistency itself restricts the form of α beyond what can currently be computed directly from operators. Thus, our perturbative treatment at the effective level provides conjectures to probe the overall consistency of the theory by comparing with results for the underlying operators.

This procedure leads us to an ansatz for the quantum corrected Hamiltonian constraint

$$H_G^Q[N] = \frac{1}{16\pi G} \int_{\Sigma} d^3x \bar{N} \alpha(\bar{p}, \delta E_i^a) \left[-6\bar{k}^2 \sqrt{\bar{p}} - \frac{\bar{k}^2}{2\bar{p}^{\frac{3}{2}}} (\delta E_j^c \delta E_k^d \delta_c^k \delta_d^j) \right. \\ \left. + \sqrt{\bar{p}} (\delta K_c^j \delta K_d^k \delta_k^c \delta_j^d) - \frac{2\bar{k}}{\sqrt{\bar{p}}} (\delta E_j^c \delta K_c^j) \right], \quad (35)$$

where $\alpha(\bar{p}, \delta E_i^a)$ is the correction function, now also depends on triad perturbations. It is important to emphasize here that the correction α coming from the quantized inverse densitized triad in general could be tensorial in nature. However, later we will see that the leading effect on perturbation dynamics comes from the background corrections, i.e. from $\bar{\alpha} = \alpha(\bar{p}, 0)$, as well as derivatives of α evaluated at the background configuration. (Note that the only background variable determining the geometry is \bar{p} , in which the corrections are expressed. This is sometimes seen as problematic since the scale factor $a = \sqrt{\bar{p}}$ can be rescaled arbitrarily in a flat isotropic model. However, the dependence of a function α in an inhomogeneous Hamiltonian constraint is through elementary fluxes whose values are determined by an underlying inhomogeneous state. The scale of corrections, too, is determined by the underlying state, resolving any apparent contradiction between the appearance of such a scale and the rescaling freedom of a flat, precisely isotropic background.)

3.2 Constraint algebra

In a canonical formulation of general relativity, classical constraints form a first class Poisson algebra, i.e. $\{C_I, C_J\} = f_{IJ}^K(A, E)C_K$, whose coefficients can in general be structure functions. It ensures that the transformations generated by the constraints are gauge and preserve the constraint surface. In other words, the evolution of phase space functions preserves the physical solution surface. To study quantum gravity effects, we have introduced a quantum correction function $\alpha(\bar{p}, \delta E_i^a)$ which depends on phase space variables. Naturally, having a new expression for the Hamiltonian constraint, there could be an anomaly term of quantum origin in the constraint algebra. Here, while $\{H_G^Q[N], H_G^Q[N']\}$ is trivial in the absence of lapse perturbations for the vector mode, a non-trivial anomaly in the algebra could occur in the Poisson bracket between $H_G^Q[N]$ and $D_G[N^a]$. This bracket turns out to be

$$\{H_G^Q[N], D_G[N^a]\} = \frac{1}{8\pi G} \int_{\Sigma} d^3x \bar{p} (\partial_j \delta N^c) \mathcal{A}_c^j, \quad (36)$$

where

$$\mathcal{A}_c^j = 3\bar{N}\bar{k}^2 \sqrt{\bar{p}} \left[\frac{\partial \alpha}{\partial(\delta E_j^c)} + \frac{1}{3\bar{p}} \frac{\partial \alpha}{\partial \bar{p}} (\delta E_k^d \delta_c^k \delta_d^j) \right]. \quad (37)$$

We see that the anomaly term contains derivative of α with respect to both \bar{p} and δE_i^a . However, as mentioned before the functional form of the correction function $\alpha(\bar{p}, \delta E_i^a)$ in terms of δE_i^a is not known while the \bar{p} -dependence can be taken to be of scalar mode form, i.e. (34) with parameters c and n fixed once an inverse triad operator is chosen. To have a consistent set of evolution equations for the vector mode we require the anomaly term to vanish i.e. $\mathcal{A}_c^j = 0$. This in turn puts restrictions on the linearized functional form of α as a function of δE_i^a :

$$\frac{\partial \alpha}{\partial(\delta E_j^c)} = -\frac{1}{3\bar{p}} \frac{\partial \alpha}{\partial \bar{p}} (\delta E_k^d \delta_c^k \delta_d^j).$$

Since α is in principle computable in the full theory, this provides important consistency checks for loop quantum gravity. At present, only the dependence $\alpha(\bar{p})$ as well as derivatives of α along diagonal components of the spatial metric are known [17]. Anomaly cancellation will then lead us to conjecture a form of derivatives $\delta\alpha/\delta(\delta E_i^a)$ along off-diagonal components of the metric which one can later compare with direct calculations once they become available.

3.3 Effective gauge invariant perturbation and its linearized equation of motion

Extrinsic curvature is derived using one of Hamilton's equations of motion. Thus, one expects the expression of extrinsic curvature as it follows from an equation of motion to change due to the quantum corrections. This incorporates an effect of quantum geometry which changes the usual differential geometric relation of extrinsic curvature corresponding

to changes of the spatial metric between different slices. One can easily compute the corrected expression for the perturbed extrinsic curvature

$$\delta K_{(a)}^i = -\frac{1}{2\alpha}(\sigma_{a,}^i + \sigma^i_{,a}) + \frac{1}{2}\bar{k}(F_{a,}^i + F^i_{,a}) \quad (38)$$

where α appears only in the first term expressed through the classical gauge invariant quantity $\sigma^a = S^a - \dot{F}^a$.

The gauge transformation of the vector perturbation F^a remains unchanged because the diffeomorphism constraint retains its classical form. Even though there is a quantum correction to extrinsic curvature one can easily see that $\sigma^a = S^a - \dot{F}^a$ is still the gauge invariant variable. Moreover with the diffeomorphism constraint being unaffected, Eq. (28) remains unchanged for the quantum background dynamics. Now using the second Hamilton's equation, one obtains an equation of motion for extrinsic curvature as follows

$$\frac{1}{\bar{\alpha}} \left[-\frac{1}{2} \frac{d}{d\eta} (\sigma_{a,}^i + \sigma^i_{,a}) - \bar{k}(\bar{\alpha} - \bar{\alpha}' \bar{p})(\sigma_{a,}^i + \sigma^i_{,a}) \right] + \mathcal{A}_{(a)}^i = 8\pi G \bar{p} \delta T^{(v)i}_{(a)} \quad (39)$$

To derive Eq. (39), we have used the corrected expression (38) of extrinsic curvature as well as an analogous expression for the background extrinsic curvature, $\bar{\alpha}\bar{k} = \dot{a}/a$. One may note here that equation (39) explicitly contain the anomaly term. Thus, the requirement of an anomaly free constraint algebra leads to the corrected equation (39) explicitly in terms of the gauge invariant variable. The presence of an anomaly, on the other hand, would make it impossible to express the equations of motion solely in terms of gauge invariant variables. Since consistency of the constraint algebra requires us to set $\mathcal{A}_a^i = 0$, closed equations for the gauge invariant perturbations follow. Nevertheless, non-trivial quantum corrections remain through $\bar{\alpha}$.

Note that the only correction function α in the Hamiltonian appears in a form multiplying the lapse function N . The correction could thus be reduced to a simple change of the lapse function and thus the time gauge. Still, the corrections are non-trivial as illustrated by the equations of motion shown here. The choice and interpretation of time is based on the line element since this determines the measurement process of co-moving geodesic observers. This behavior is not changed by the appearance of a correction function α in the Hamiltonian constraint even if it always appears in combination with N . Even with a corrected Hamiltonian constraint we are still referring to the same form of conformal time, but fields evolve differently as given by the corrected constraint. Then, also observable implications of the quantum corrections are possible.

4 Quantum dynamics: Holonomy corrections

In addition to corrections in coefficients of the constraint due to inverse powers of densitized triad components, there are corrections which resemble higher curvature terms in an effective action. While these corrections would be dominant in purely isotropic models by virtue of the large matter energy density in a macroscopic universe, they are sub-dominant

in inhomogeneous situations [37]. Moreover, as we will see they do not provide much of a structural change to the equations. The Hamiltonian constraint operator is formulated in terms of holonomies rather than connection or extrinsic curvature components. Since these objects are non-linear as well as (spatially) non-local in connection components, they provide higher order and higher spatial derivative terms. Higher time derivatives, as they would also be provided by higher curvature terms, do not arise in this way but rather through the coupling of fluctuations and higher moments of a quantum state to the expectation values [26, 27]. The full effective constraint including all these terms has not been derived yet. In this section we therefore focus on an analysis of higher order terms only.

For an isotropic model sourced by a massless, free scalar field such higher order terms turn out to be the only corrections, and there are no higher time derivatives [38]. The exact effective Hamiltonian can then be obtained by simply replacing the background Ashtekar connection $\gamma\bar{k}$ by $\bar{\mu}^{-1} \sin \bar{\mu}\gamma\bar{k}$, as it was also seen in numerical studies [39]. The parameter $\bar{\mu}$ depends on the quantization scheme and may be a function of \bar{p} . Just as with the parameter c in (34), we will see that the freedom is constrained by anomaly cancellation. To study the effects of the background dynamics on inhomogeneous perturbations, we similarly substitute the appearance of \bar{k} in the classical Hamiltonian by a general form $(m\bar{\mu})^{-1} \sin m\bar{\mu}\gamma\bar{k}$ where m is an integer. (This parameter is kept free because different factors of sines and cosines combine from the full constraint to result in this term. It can be constrained by looking at detailed properties of the underlying operator, but also by consistency requirements as we will see shortly.) With this prescription, one can write down expression for the corrected Hamiltonian constraint

$$H_G^Q[N] = \frac{1}{16\pi G} \int_{\Sigma} d^3x \bar{N} \left[-6\sqrt{\bar{p}} \left(\frac{\sin \bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma} \right)^2 - \frac{1}{2\bar{p}^{\frac{3}{2}}} \left(\frac{\sin \bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma} \right)^2 (\delta E_j^c \delta E_k^d \delta_c^k \delta_d^j) \right. \\ \left. + \sqrt{\bar{p}} (\delta K_c^j \delta K_d^k \delta_k^c \delta_j^d) - \frac{2}{\sqrt{\bar{p}}} \left(\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma} \right) (\delta E_j^c \delta K_c^j) \right]. \quad (40)$$

Here we have required that the effective Hamiltonian (40) has a homogeneous limit in agreement with what has been used in isotropic models. This fixes the parameter m to equal one in the first two terms. The parameter for the last term as chosen here is the one which leads to an anomaly-free constraint algebra.

Although we write explicit sines in this expression, and thus arbitrarily high powers of curvature components, it is to be understood only as a short form to write the leading order corrections. Higher orders are supplemented by further, yet to be computed higher curvature quantum corrections. The expressions are thus reliable only when the argument of the sines is small, which excludes the bounce phase itself. (Such sine corrections can be used throughout the bounce phase only for exactly isotropic models sourced by a free, massless scalar [38, 40], but not in the presence of a matter perturbation [41] or anisotropies and inhomogeneities.)

4.1 Constraint algebra

Again, a non-trivial anomaly in the algebra can occur between the Poisson bracket between $H_G^Q[N]$ and $D_G[N^a]$

$$\{H_G^Q[N], D_G[N^a]\} = \frac{\bar{N}}{\sqrt{\bar{p}}} \left(\bar{k} - \frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma} \right) D_G[N^a] + \frac{1}{8\pi G} \int_{\Sigma} d^3x \bar{p} (\partial_c \delta N^j) \mathcal{A}_j^c, \quad (41)$$

where

$$\mathcal{A}_j^c = \frac{\bar{N}}{\sqrt{\bar{p}}} \left[\bar{p} \frac{\partial}{\partial \bar{p}} \left(\frac{\sin \bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma} \right)^2 + \left(\frac{\sin \bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma} \right)^2 - \bar{k}^2 \right] \left(\frac{\delta E_j^c}{\bar{p}} \right). \quad (42)$$

One may easily check here that $\bar{\mu} \sim 1/\sqrt{\bar{p}}$ leads to an anomaly free algebra up to order \bar{k}^4 . This is in accordance with the result of arguments put forward recently in purely isotropic models [42]. From an inhomogeneous perspective, the behavior $\bar{\mu} \sim 1/\sqrt{\bar{p}}$ reflects the fact that the fundamental Hamiltonian creates new vertices when acting on a graph state such that the number of vertices increases linearly with volume [37, 43]. This suggests a tight relation between anomaly freedom at the effective level and properties such as the creation of new vertices by a fundamental Hamiltonian constraint.

4.2 Effective linearized equation

Hamilton's equations governing the background dynamics are given by

$$\dot{\bar{p}} = 2\bar{p} \left(\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma} \right) \quad (43)$$

and

$$\dot{\bar{k}} = -\frac{\bar{N}}{\sqrt{\bar{p}}} \left[\frac{1}{2} \left(\frac{\sin \bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma} \right)^2 + \bar{p} \frac{\partial}{\partial \bar{p}} \left(\frac{\sin \bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma} \right)^2 \right] + \frac{8\pi G}{3V_0} \left(\frac{\partial \bar{H}_m}{\partial \bar{p}} \right). \quad (44)$$

Since the diffeomorphism constraint is assumed to remain unaffected, the gauge transformation of vector functions F^a remains unchanged. However, a correction to the extrinsic curvature expression now leads to a new expression for the gauge invariant variable

$$\sigma^a = S^a - \dot{F}^a + \bar{k} \left(1 - \frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma\bar{k}} \right) F^a. \quad (45)$$

The expression of perturbed extrinsic curvature in terms of the gauge invariable variable remains unchanged, though,

$$\delta K_{(a)}^{i} = -\frac{1}{2}(\sigma_{a,}^{i} + \sigma^i_{,a}) + \frac{1}{2}\bar{k}(F_{a,}^{i} + F^i_{,a}). \quad (46)$$

Now using again Hamilton's equation, one obtains an equation of motion for extrinsic curvature as follows

$$-\frac{1}{2} \frac{d}{d\eta} (\sigma_{a,}^{i} + \sigma^i_{,a}) - \frac{1}{2} \bar{k} \left(1 + \frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma\bar{k}} \right) (\sigma_{a,}^{i} + \sigma^i_{,a}) + \mathcal{A}_{(a)}^i = 8\pi G \bar{p} \delta T^{(v)(i)}_{(a)}. \quad (47)$$

As before, an anomaly-free constraint algebra, requiring $\mathcal{A}_a^i = 0$, leads to a quantum corrected equation entirely in terms of the gauge invariant variable.

5 Rate of change of vector perturbations

We now have equations that govern the dynamics of vector mode perturbations including quantum corrections. For simplicity, we consider the situation where anisotropic stress is absent, which is the context of [25].

5.1 Classical dynamics

Let us recall the key feature of classical vector mode perturbations. In the absence of anisotropic stress, the right hand side of equation (31) vanishes. The classical background extrinsic curvature is related to the time derivative of the scale factor a as $\bar{k} = \dot{a}/a$. It is also convenient to decompose the amplitude of perturbations in terms of their Fourier modes σ_k^i . With these simplifications, equation (31) leads to the rate of change

$$\frac{d \log \sigma_k^i}{d \log a} = -2 \quad (48)$$

for Fourier modes. Thus, any vector mode grows as $\sigma_k^i \sim a^{-2}$ in a contracting phase, and correspondingly decays in an expanding phase. This is independent of the background matter content except for the assumed absence of anisotropic stress. Thus, a quantum correction of the background matter sector alone would not help in taming the growth of vector perturbations. Intuitively, one expects that it is required to have a modified gravity sector in order to have any modification of the growth rate. In the next sub-section we consider the dynamics in the presence of quantum corrections to the gravitational Hamiltonian.

5.2 Quantum corrections: Inverse triad

As in the classical case, we consider the situation where anisotropic stress is absent. Then using the equation of motion $\bar{\alpha}\bar{k} = \dot{a}/a$ for the background, one can write Eq. (39) for Fourier modes σ_k^i of vector perturbations in the form

$$\frac{d \log \sigma_k^i}{d \log a} = -2 \left(\frac{\bar{\alpha} - \bar{\alpha}' \bar{p}}{\bar{\alpha}} \right) . \quad (49)$$

For $\bar{\alpha} = 1$ quantum corrections are switched off and we obtain the classical result. Now using the generic form of $\bar{\alpha}$ as in (34) with an approach to one from above (i.e. $c > 0$), it is easy to see that

$$\left(\frac{\bar{\alpha} - \bar{\alpha}' \bar{p}}{\bar{\alpha}} \right) = 1 + nc \left(\frac{\ell_P^2}{\bar{p}} \right)^n > 1 . \quad (50)$$

Thus, the decay rate of vector mode perturbations is slightly higher compared to that of a classical scenario. In other words, in the contracting phase, the correction coming from inverse powers of the densitized triad in background dynamics causes vector perturbations to grow even faster, though only slightly, than in the classical situation. The quantum

correction depends inversely on volume, i.e. it becomes stronger in the smaller volume regime. Using non-perturbative corrections in $\bar{\alpha}$ for small densitized triads, a decrease of the rate is indicated since $\bar{\alpha}$ falls below one in this regime and has $\bar{\alpha}' > 0$. However, for such small scales the perturbation theory of inhomogeneities is less reliable and a suppression of the decay rate on very small scales can, at present, at best be taken as an indication.

5.3 Quantum corrections: Holonomies

As before, we consider the situation where anisotropic stress is absent. Now, the corrected equation for Fourier modes σ_k^i of vector perturbations, after dividing (47) by (43), is

$$\frac{d \log \sigma_k^i}{d \log a} = - \left(1 + \frac{2\bar{\mu}\gamma\bar{k}}{\sin 2\bar{\mu}\gamma\bar{k}} \right). \quad (51)$$

Here, quantum corrections disappear for $\bar{\mu} \rightarrow 0$. Again, the right hand side is less than -2 and thus vector modes grow more strongly in a contracting phase. If the behavior is extrapolated to the bounce phase, the growth rate in a contracting universe becomes even larger and would diverge at the bounce where $\cos(\bar{\mu}\gamma\bar{k})$ is zero. This indicates a breakdown of the perturbation scheme and the need to include higher order terms as the bounce is approached. It should also be emphasized here that $\bar{\mu} \sim 1/\sqrt{p}$ leads to anomaly free Poisson algebra only up to order \bar{k}^4 . So the anomaly term becomes significant near bounce phase that makes the analysis less reliable there.

6 Discussions

In the absence of anisotropic stress, gauge-invariant vector perturbations classically grow as a^{-2} in the contracting phase. Such a growth of vector perturbations indicates a possible violation of homogeneity assumptions in smaller volume regime, indicated by the breakdown of classical perturbation theory. Thus, the growth of vector perturbations may pose significant problems in particular for bouncing cosmologies which are invariably associated with a contracting phase but often have been derived only under the assumption of homogeneity. In these models, conclusions regarding bounces are drawn based on the homogeneity assumption. Naturally, a growth of vector perturbations can question the robustness of such bounce scenarios by questioning the validity of the homogeneity assumption itself at smaller volume, given that bounces are typically more difficult to realize when inhomogeneities are taken into account; see [44, 38]. Thus, it is an important issue to study the dynamics of vector mode perturbations in the cosmological context.

In this paper, we have presented a systematic derivation of gauge-invariant vector perturbation equations to linear order in Hamiltonian cosmology based on Ashtekar variables. We have only considered a spatially flat Friedmann–Robertson–Walker background as this is the case of most interest in cosmology. Hamiltonians and equations of motion are technically more complicated in the presence of spatial curvature and are still being worked out. Quantum corrections are, however, analogous in spatially curved backgrounds and we do

not expect our results to change significantly in those cases. Specifically, we have studied the effects of two particular types of quantum correction, inverse triad and holonomy corrections, on the dynamics of vector perturbations in large volume regimes. For each type, we have shown that in a contracting phase the growth rate of vector mode perturbations is slightly stronger compared to the classical situation due to quantum effects. This quantum correction is small as one expects. Although there are quantization ambiguities, the sign of corrections to the growth rate seems robust. A reduction of the growth rate is indicated only in regimes of non-perturbative corrections of inverse powers. For such a reduction to be realized, even if it remains true under a more careful perturbation analysis, one would have to enter the deep Planck regime. Moreover, if one starts with a large classical universe as initial configuration, such non-perturbative quantum effects will become relevant only after long evolution times. In general, due to the growth of the vector mode one will eventually have to use higher than linear orders in perturbative inhomogeneities which we have not included in this paper. This by itself may well change some of the conclusions about the bounce phase independently of quantum effects in the evolution of inhomogeneities.

Another important issue touched in this paper is that of potential anomalies in the quantum constraint algebra. We started with a general but unspecified form of a quantum correction function α or higher order terms, including inhomogeneity. The functional form of α as a function of the background variable \bar{p} is known. However, its functional dependence on the perturbed densitized triad δE_i^a (which is purely off-diagonal for vector modes) is unknown due to the lack of a systematic derivation of such corrections from the full theory. As we have observed, requiring anomaly cancellation in the modified constraint algebra restricts the functional dependence of quantum correction functions such as α on off-diagonal triad components. It would be interesting to see whether such a restriction is satisfied by a systematically derived quantum correction function from the full theory. As a key result, we have observed the possibility of non-trivial quantum corrections while preserving anomaly freedom. The classical constraint algebra for vector modes is rather trivial, but is much more restrictive for scalar modes for which the calculations here show the guiding principle.

Acknowledgements

We would like to thank Mikhail Kagan for discussions. This work was supported in part by NSF grants PHY0653127, PHY0554771 and PHY0456913.

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